

CDM Distribution in the Eddington Theory.

# New Velocity Distribution for Cold Dark Matter in the Context of the Eddington Theory

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## ABSTRACT

Exotic dark matter together with the vacuum energy (associated with the cosmological constant) seem to dominate the Universe. Thus its direct detection is central to particle physics and cosmology. Supersymmetry provides a natural dark matter candidate, the lightest supersymmetric particle (LSP). One essential ingredient in obtaining the direct detection rates is the density and velocity distribution of the LSP. The detection rate is proportional to this density in our vicinity. Furthermore, since the rates are expected to be very low, one should explore the two characteristic signatures of the process, namely the modulation effect, i.e. the dependence of the event rate on the Earth's motion and the correlation of the directional rate with the motion of the sun. Both of these crucially depend on the LSP velocity distribution. In the present paper we study simultaneously density profiles and velocity distributions based on the Eddington theory.

*Subject headings:* Cosmology:Eddington theory, velocity profiles, rotational curves-  
Cold Dark Matter:velocity distribution, direct detection rates.

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## 1. Introduction

In recent years the consideration of exotic dark matter has become necessary in order to close the Universe (Jungman et al. 1996). The COBE data (Smoot et al. 1992) suggest that CDM (Cold Dark Matter) component is at least 60% (Gawser et al. 1988; Gross et al. 1998) of the total mass. On the other hand evidence from two different teams, the High- $z$  Supernova Search Team (Riess et al. 1998) and the Supernova Cosmology Project (Somerville et al. 2002), (Perlmutter et al. 1999; Perlmutter et al. 1997) suggests that the Universe may be dominated by the cosmological constant  $\Lambda$ . Thus the situation can be adequately described by a baryonic component  $\Omega_B = 0.1$  along with the exotic components  $\Omega_{CDM} = 0.3$  and  $\Omega_\Lambda = 0.6$ . In another analysis Turner (Turner 1990) gives  $\Omega_m = \Omega_{CDM} + \Omega_B = 0.4$ . Since the non exotic component cannot exceed 40% of the CDM (Jungman et al. 1996), (Alcock et al. 1995), there is room for the exotic WIMP's (Weakly Interacting Massive Particles). In fact the DAMA experiment (Bernabei et al. 1996) has claimed the observation of one signal in direct detection of a WIMP, which with better statistics has subsequently been interpreted as a modulation signal (Bernabei et al. 1998; Bernabei et al. 1999).

In the most favored scenario of supersymmetry the LSP can be simply described as a Majorana fermion, a linear combination of the neutral components of the gauginos and Higgsinos (Jungman et al. 1996; Vergados 1990; Gomez and Vergados 2001; Gomez et al. 2000a,b; Gomez and Vergados 2001; Anrowit and Nath 1995, 1996; Bottino et al. 1997; Bednyakov et al. 1994).

Since this particle is expected to be very massive,  $m_\chi \geq 30\text{GeV}$ , and extremely non relativistic with average kinetic energy  $T \leq 100\text{KeV}$ , it can be directly detected (Vergados 1996; Spira et al. 1995; Kosmas and Vergados 1997) mainly via the recoiling of a nucleus (A,Z) in elastic scattering.

In order to compute the event rate one needs the following ingredients:

1) An effective Lagrangian at the elementary particle (quark) level obtained in the framework of supersymmetry as described, e.g., in (Jungman et al. 1996; Anrowit and Nath 1995, 1996; Bottino et al. 1997; Bednyakov et al. 1994).

2) A procedure in going from the quark to the nucleon level, i.e. a quark model for the nucleon. The results depend crucially on the content of the nucleon in quarks other than u and d. This is particularly true for the scalar couplings as well as the isoscalar axial coupling (Drees and Nojiri 1993b,c; Cheng 1988, 1989).

3) Compute the relevant nuclear matrix elements (Ressel et al. 1993; Divari et al. 2000)

using as reliable as possible many body nuclear wave functions. The situation is a bit simpler in the case of the scalar coupling, in which case one only needs the nuclear form factor.

4) The LSP density and velocity distribution. Among other other things, since the detection rates are expected to be very small, the velocity distribution is crucial in exploiting the characteristic experimental signatures provided by the reaction, namely: a) the modulation of the event rates due to the earth’s revolution around the sun (Vergados 1998, 1999)–(Vergados 2000) and b) the correlation of the rates with the Sun’s direction of motion in directional experiments, i.e. experiments in which the direction of the recoiling nucleus is observed (Vergados 1990; Buckland et al. 2000). To obtain the right density and velocity distributions is the purpose of the present paper.

In the past various velocity distributions have been considered. The most popular one is the isothermal Maxwell-Boltzmann velocity distribution with  $\langle v^2 \rangle = (3/2)v_0^2$  where  $v_0$  is the velocity of the sun around the galaxy, i.e.  $220 \text{ km/s}$ . Extensions of this M-B distribution were also considered, in particular those that were axially symmetric with enhanced dispersion in the galactocentric direction (Drucker et al. 1986; Vergados 2000). In such distributions an upper cutoff  $v_{esc} = 2.84v_0$  was introduced by hand.

Non isothermal models have also been considered. Among those one should mention the late infall of dark matter into the galaxy, i.e caustic rings (Sikivie 1999(, 1998; Vergados 2001a; Green 2001; Gelmini and Gondolo 2001), and dark matter orbiting the Sun (Copi et al. 1999).

The correct approach in our view is to consider the Eddington approach (Eddington 1916), i.e. to obtain both the density and the velocity distribution from a distribution from a mass distribution, which depends both on the velocity and the gravitational potential. This approach has been extensively studied by Merritt (Merritt 1985) and recently applied to dark matter by Ullio and Kamionkowski(Ullio and Kamiokowski 2001)

## 2. Density Profiles

As we have seen in the introduction the matter distribution can be given as follows

$$dM = 2\pi f(\Phi(r), v_r, v_t) dx dy dz v_t dv_t dv_r \quad (1)$$

where the function  $f$  the distribution function, which depends on  $r$  through the potential  $\Phi(r)$  and the tangential and radial velocities  $v_t$  and  $v_r$  (we assume axial symmetry in velocity space). Thus the density of matter  $\rho$  satisfies the equation

$$d\rho = 2\pi f(\Phi(r), v_r, v_t) v_t dv_t dv_r \quad (2)$$

It is more convenient instead of the velocities to use the total energy  $E$  and the angular momentum  $J$  via the equations

$$J = v_t r \quad , \quad 2E = v_r^2 + \frac{J^2}{r^2} + 2 \Phi(r) \quad (3)$$

We thus find

$$\rho = \frac{2\pi}{r^2} \int \int \frac{f(E, J) J}{\sqrt{2(E - \Phi(r)) - J^2/r^2}} dJ dE \quad (4)$$

The limits of integration for  $E$  are from  $\Phi$  to 0 and for  $J$  from 0 to  $[2r^2(E - \Phi(r))]^{1/2}$ .

Following Eddington we will choose a distribution function of the form

$$f(E, J) = (-2E)^\lambda \quad (5)$$

( $E$  is negative for a bound system), where  $\lambda$  is a parameter, which will depend on the type of matter. With this choice of the distribution function it is quite straightforward to find the relationship between the density  $\rho$  and the potential. The result is

$$\rho = 2^{\lambda+3/2} \pi |\Phi(r)|^{\lambda+3/2} \beta(\lambda + 1, 3/2) \quad (6)$$

with

$$\beta(a, b) = \frac{\Gamma(a) \Gamma(b)}{\Gamma(a + b)} \quad (7)$$

The above distribution function is isotropic. Following Merritt (Merritt 1985) we can introduce an anisotropy by modifying the distribution function as follows:

$$f(E, J) = (-2E)^\lambda (1 \pm \frac{J^2}{r_a^2}) \quad (8)$$

Instead of the parameter  $r_a$  we find it convenient for our applications later on (see below) to adopt the more recent conventions and write the above equation as follows:

$$f(E, J) = (-2E)^\lambda [1 + \alpha_s \frac{J^2}{(r_s v_0)^2}] \quad (9)$$

where  $r_s$  is the position of the sun,  $v_0$  its velocity around the center of the galaxy and  $\alpha_s$  the asymmetry parameter. Proceeding as above we find that this induces a correction to the density of the form:

$$\Delta\rho = \frac{4}{3} 2^{\lambda+3/2} \pi |\Phi(r)|^{\lambda+5/2} \beta(\lambda + 1, 5/2) \frac{\alpha_s r^2}{v^2 r_s^2} \quad (10)$$

The above equations hold for both ordinary matter ( $\lambda = 7/2$ ) and dark matter ( $\lambda = 1/2$ ).

Combining Eqs (6) and (10) we get

$$\rho(r) = \rho_0 \frac{\psi(x)}{\psi(x_s)} \quad (11)$$

$$\psi(x) = \frac{\Phi^{\lambda+3/2}(x)}{\Phi^{\lambda+3/2}(x_s)} \left[ 1 + \frac{4}{3} a \frac{\beta(\lambda+1, 5/2) x^2 \Phi(x)}{\beta(\lambda+1, 3/2) x_s^2 \Phi(x_s)} \right] \quad (12)$$

The potential  $\Phi(x)$  is expressed in the dimensionless parameter  $x = r/b$ , where  $b$  is the outer radius of the halo.  $\rho_0$  is the density in our vicinity  $x_s = r_s/b$ ,  $\rho_0 = \rho(x_s)$ , and  $a = \alpha_s \frac{\Phi(x_s)}{v_0^2} = \pm \frac{\Phi(x_s) b^2}{r_a^2}$ . Since the scale of the potential appears only via the parameter  $a$  one, in principle, could have two  $a$  parameters, one for Matter ( $a_m$ ) and one for dark matter ( $a_{dm}$ ). In the present work we will assume that they are equal. We remind the reader that  $\alpha_s$  is the asymmetry parameter, which is related to the Merritt parameter via the equation  $\alpha_s = \pm v_0^2 (r_s/r_a)^2$ .

### 3. Rotational Velocities

In obtaining the rotational velocities we will treat  $a$  as a phenomenological parameter. In particular we will consider two values for  $a$ , namely  $a = 0, -1$ .

In this work we will consider potentials of the form:

$$\Phi(x) = \frac{\Phi(0)}{(1+x^2)^{n/2(\lambda+3/2)}} \quad (13)$$

with  $n=1,2$  and two values of  $\lambda$ , i.e  $\lambda = 1/2$  (dark matter) and  $\lambda = 7/2$  for ordinary matter. With the above ingredients we will study the rotational velocities.

i) Dark matter.

The rotational velocity is normalized to its value at infinity, here  $x = b$ . Thus we write.

$$\left( \frac{v(x)}{v(\infty)} \right)^2 = g_{dark}(x) \quad , \quad g_{dark}(x) = \frac{g_{dm}(x)}{g_{dm}(b)} \quad (14)$$

For  $n = 1$  and  $\lambda = 1/2$  we get for  $x < b$

$$g_{dm}(x) = 1 - \frac{\tan^{-1}(x)}{x} + \frac{2}{3} a \left[ -\frac{x^2}{\sqrt{(1+x^2)}} + \frac{3}{2} (\sqrt{(1+x^2)} - \frac{\ln(x + \sqrt{(1+x^2)})}{x}) \right] \quad (15)$$

For  $n = 2$  we get for  $x < b$

$$g_{dm}(x) = \frac{1}{2} \left( \frac{\tan^{-1}(x)}{x} - \frac{1}{1+x^2} \right) + \frac{2}{3} a \left[ -\frac{x^2}{(2(1+x^2))^2} + \frac{3}{8} \left( \frac{\tan^{-1}(x)}{x} - \frac{1}{(1+x^2)} \right) \right] \quad (16)$$

For  $x > b$  we get for both cases

$$\left(\frac{v(x)}{v(\infty)}\right)^2 = \frac{b}{x} g_{dm}(b) \quad (17)$$

In the above notation we have for dark matter

$$v(\infty))^2 = 4\pi G_N \rho_0 \frac{b^3}{r_s} \frac{g_{dark}(1)}{g_{dark}(x_s)} \quad (18)$$

The rotational velocities associated with dark matter are exhibited in Fig. 1. ii) Ordinary matter.

In this case we normalize the matter density so that the rotational velocity at the radius of the galaxy,  $x = x_s$ , is proportional to the dark matter velocity at some distance  $d$  with proportionality constant  $C_{mdm}$ . Thus

$$\frac{v^2(x)}{v^2(x_s)} = g_{matter}(x) \quad , \quad g_{matter}(x) = C_{mdm} g_{dm}(x_d) \frac{g_m(x)}{g_m(x_s)} \quad (19)$$

where  $d$  some suitable distance, here  $d = x_s$ . We treat  $C_{mdm}$  as a phenomenological parameter, which can be determined from the observed rotational velocity curves.

We now consider two cases, namely a spiral galaxy and a spherical one. For  $x < x_s$  we write

$$g_m(x) = g(x) \quad (20)$$

For a spiral galaxy (disk) we find

$$g(x) = \frac{1}{3x} \left[1 - \frac{1}{(1+x^2)^{3/2}}\right] + \frac{1}{3} a \left[\frac{1}{4(1+x^2)^2} - \frac{1}{2(1+x^2)} + \frac{1}{4}\right] \quad (21)$$

On the other hand for a spherical galaxy we find

$$g(x) = \frac{1}{3} \frac{x^2}{(1+x^2)^{3/2}} + \frac{1}{3} a \left[-\frac{x^2}{4(1+x^2)^2} + \frac{3}{8} \left(-\frac{1}{1+x^2} + \frac{\tan^{-1}(x)}{x}\right)\right] \quad (22)$$

For  $x > x_s$  for either type of galaxy we find:

$$g_m(x) = \frac{x_s}{x} g(x_s) \quad (23)$$

We note that the rotational velocity with  $\alpha_s = 0$  has a maximum around  $x = 0.8$  and  $x = 1.4$  for spiral and spherical galaxy respectively. This distance is smaller than the radius of the galaxy and explains why the maximum of the rotational velocity curves exhibited in Figs 2 occurs at distances less than  $x_s$ . The overall rotational velocity is conveniently normalized

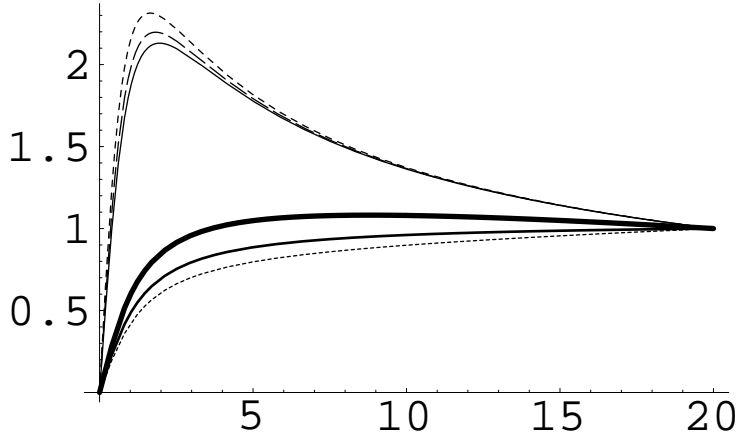


Fig. 1.— The rotational velocity vs  $x$  associated with dark matter only, normalized to unity at  $x = 1$ . The thick solid line, the medium thickness solid line, the very short dashed line, the short dashed line, the very fine solid line and the long dashed line correspond respectively to  $(-0.05, 1)$ ,  $(0, 1)$ ,  $(0.05, 1)$ ,  $(-0.5, 2)$ ,  $(0, 2)$ ,  $(0.05, 2)$  with the first number of the pair indicating  $\alpha_s$  while the second specifying  $n$ .

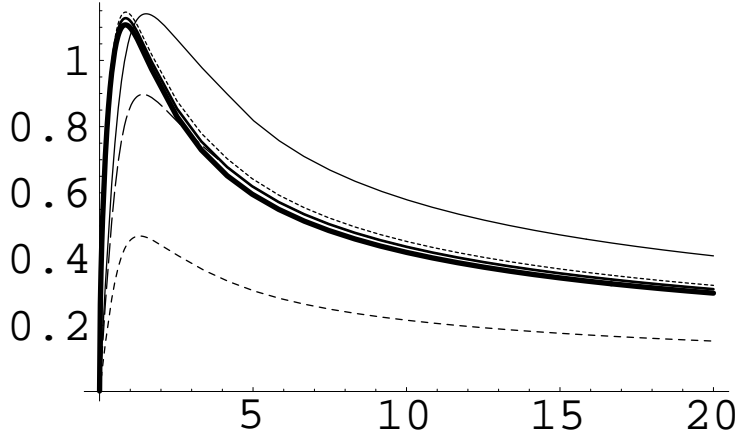


Fig. 2.— The rotational velocity associated with only ordinary matter. It is normalized so that at the galactic radius,  $x = x_s = b/5$ , ordinary matter density is 2.5 times the dark matter density at that point. There is no big difference between a spiral galaxy (thick solid line) and a spherical galaxy (thin line).

to its maximum value at  $x = b$  and is given by:

$$\left(\frac{v(x)}{v(\infty)}\right)^2 = \frac{g_{matter}(x) + g_{dark}(x)}{g_{matter}(b) + g_{dark}(b)} \quad (24)$$

To summarize the rotational velocities depend on the parameters  $a$ ,  $C_{mdm}$  and  $x_d$ .  $\rho_0$  has been absorbed in the normalization of the rotational velocities. If, however, we choose  $x_d = x_s$ , then  $C_{mdm}$  is just the ratio of the densities at  $x = x_s$ . We should stress that it is only in the correction term that the scale of the potential enters. For  $C_{mdm} = 5/2$  the rotational velocities are exhibited in Fig. 3, while for  $C_{mdm} = 1$  and 0.5 are shown in Figs 4 and 5.

The spike in Figs 3 and 4 is due to ordinary matter. We see that a good fit to the data comes from the  $n = 1$  case, independently of the asymmetry parameter  $\alpha_s$ . In fact the smaller value of  $C_{mdm}$  seems to be in better agreement with the data, see, e.g., the recent review (Jungman et al. 1996).

#### 4. Velocity distribution with respect to the galactic center

The above density via the Eddington formula leads to a velocity distribution of the form:

$$F(v, r) = \propto \rho_0 [-2\Phi(r) - v^2]^\lambda (1 + \alpha_s \frac{v_t^2}{v_0^2}) \quad (25)$$

The above velocities and the distance  $r$  refer to the center of the galaxy. We note in the context of the Eddington theory the velocity distribution cannot be Maxwellian. For a given distance it goes to zero at the boundaries of the corresponding ellipsoid. It is customary to consider the value of the above distribution in our vicinity,  $r = r_s$ . This way it reduces to the product of the local density and the velocity distribution. The latter is

$$f(v) = N [-2\Phi(r_s) - v^2]^\lambda (1 + \alpha_s \frac{v_t^2}{v_0^2}) \quad (26)$$

where  $N$  is a normalization constant, which depends on  $\lambda$ ,  $\alpha_s$  and  $v_0$ . The above notation was introduced to make the last equation coincide with the standard expression when the function  $f$  is chosen to be Maxwellian, i.e.

$$\text{Exp}(-\frac{v_s^2 + (1 - \alpha_s)v_t^2}{v_0^2}) \rightarrow \text{Exp}(-\frac{v^2}{v_0^2})(1 + \alpha_s \frac{v_t^2}{v_0^2})$$

for sufficiently small  $\alpha_s$ . In this limit we see that  $\alpha_s$  coincides with the parameter  $-\lambda$  of Vergados (Vergados 2000) *et al* Drucker et al. (1986) (in the present work  $\lambda$  is used for another purpose). It is straightforward to find that the normalization factor  $N$  is given by

$$N^{-1}(\alpha_s, y_m) = 2\pi(2|\Phi(x_s)|)^{\lambda+3/2} \beta(\lambda+1, 3/2) [1 + \frac{4}{3} \alpha_s \frac{\beta(\lambda+1, 5/2)}{\beta(\lambda+1, 3/2)}] \quad (27)$$



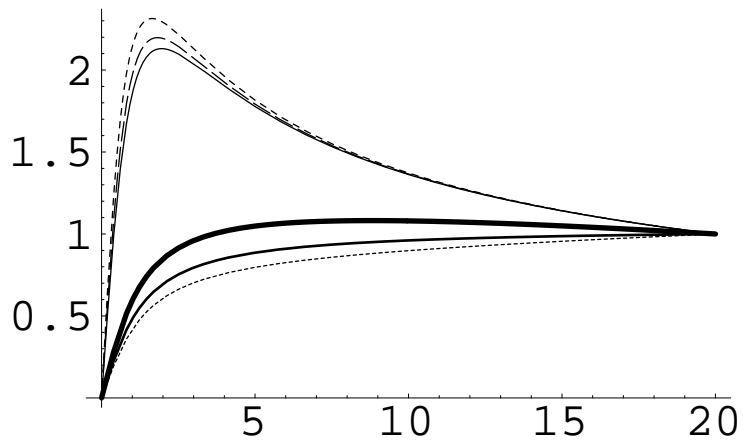


Fig. 3.— The rotational velocity associated with both ordinary matter and dark matter in the notation of Fig. 1. Only spiral galaxies were considered with considered with  $C_{mdm} = 5/2$  at  $r = r_s$ . The curves were normalized to unity at  $x=b$ .

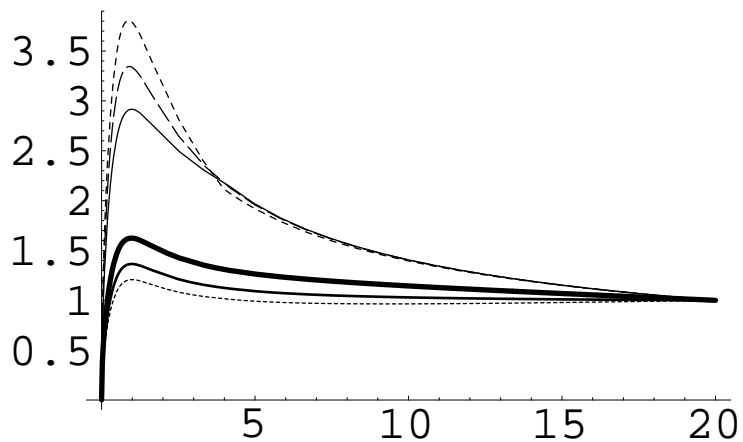


Fig. 4.— The same as in Fig. 3 for  $C_{mdm} = 1$ .

In the special case of dark matter ( $\lambda = 1/2$ ) it becomes

$$N^{-1}(\alpha_s, y_m) = \frac{\pi^2}{4}(-2\Phi(x_s))^2 \left[1 + \frac{2}{3}\alpha_s \frac{-\Phi(x_s)}{v_0^2}\right] \quad (28)$$

From the above formulas we see that the velocity of dark matter with respect to the galactic center ranges from 0 to a maximum speed  $v_m = (-2\Phi(x_s))^{1/2}$ . Since the potential  $\Phi(r_s)$  is expected to be the same both for ordinary matter and dark matter, in spite of the fact the the corresponding densities differ, we assume that  $v_m^2 = (1/2)v_{esc}^2$ , where  $v_{esc} = 2.84 v_0$  is the escape velocity. Thus, since the distribution function must remain positive, if  $\alpha_s < 0$  its absolute value cannot exceed  $\frac{v_0^2}{v_m^2} = 0.27$ . This is indeed a very useful constrain, since in the traditional analysis with only axially symmetric Gaussian distribution it leads to enhanced dispersion in the galactocentric direction, a phenomenologically preferred result (Drucker et al. 1986). In the case of positive asymmetry parameter no such constraint exists, but one does not expect the correction term to be too large. In any case  $a$  can be constrained by requiring the above rotational velocities to remain positive everywhere.

From then on one proceeds in the usual way to obtain the velocity distribution with respect to the laboratory.

## 5. Velocity distribution with respect to the laboratory

For this transformation one needs the velocity of the sun around the galaxy  $v_0 = 220 Km/s$ , a fraction of the escape velocity, which is  $v_{esc} = 625 Km/s = 2.84 v_0$  (Drucker et al. 1986).

It is convenient to choose as polar z-axis in the the direction of the disc's rotation, i.e. in the direction of the motion of the the sun, the y-axis is perpendicular to the plane of the galaxy and the x-axis is in the radial direction. Since the axis of the ecliptic (Kosmas and Vergados 1997). lies very close to the  $y, z$  plane the velocity of the earth around the sun is given by

$$v_E = v_0 \hat{\mathbf{z}} + v_1 = v_0 \hat{\mathbf{z}} + v_1 (\sin \alpha \hat{\mathbf{x}} - \cos \alpha \cos \gamma \hat{\mathbf{y}} + \cos \alpha \sin \gamma \hat{\mathbf{z}}) \quad (29)$$

where  $\alpha$  is the phase of the earth's orbital motion,  $\alpha = 2\pi(t - t_1)/T_E$ , where  $t_1$  is around second of June and  $T_E = 1year$ . The magnitude of the Earth's velocity is much smaller than that of the sun, i.e.  $\delta_1 = 2v_1/v_0 = 0.27$ . The velocity of the earth around its own axis is smaller and it can safely be neglected.

One can now express the above distribution in the laboratory frame by writing  $v' = v + v_E$ , where the prime indicates the velocity with respect to the center of the galaxy. We

thus find

$$f(y, \theta, \phi) = N [y_m^2 - Y(y, \theta, \phi)]^\lambda [1 + \alpha_s(Y(y, \theta) - (y \sin \theta \cos \phi - \frac{\delta_1}{2} \sin \alpha)^2)] \quad (30)$$

where  $N$  is given by Eq. 28,  $y_m = v_m/v_0$  and

$$\begin{aligned} Y(y, \theta, \phi) = & 1 + \frac{\delta_1^2}{4} + y^2 + 2y \cos \theta + \delta_1 [y \cos \theta \cos \alpha \sin \gamma \\ & - y \sin \theta \sin \phi \cos \alpha \cos \gamma + y \sin \theta \cos \phi \sin \alpha] \end{aligned} \quad (31)$$

with  $y = \frac{v}{v_0}$  and  $y_m = \frac{y_{esc}}{\sqrt{2}}$ . In the conventional axially symmetric Gaussian velocity distribution it is given by

$$\begin{aligned} f(y, \theta, \phi) = & \frac{N(\alpha_s, y_{esc})}{\pi \sqrt{\pi}} \text{Exp}[-(\alpha_s + 1)Y(y, \theta, \phi)] \\ & + \alpha_s (y \sin \theta \cos \phi - \frac{\delta_1}{2} \sin \alpha)^2] \end{aligned} \quad (32)$$

In this case  $0 \leq y \leq y_{esc}$ , but the upper cutoff is introduced here artificially. The normalization here is defined so that  $N(\alpha_s = 0, y_{esc} \rightarrow \infty) = 1$

In the present velocity distribution, unlike the Gaussian case, one cannot approximate the distribution by a power series in  $\delta_1$ . The reason is that  $\lambda$  is non integer, as a matter of fact for dark matter  $\lambda = 1/2$ . So there may be threshold problems when the argument of the square root goes to zero.

The detection rate in direct dark matter experiments is obtained by convoluting the the relevant cross section with the above velocity distribution. If the dark matter candidate is the LSP (LSP is the Lightest Supersymmetric Particle), the  $\alpha$ -dependence of the above distribution, present only when  $\delta_1 \neq 0$ , gives rise to the modulation effect, i.e. the dependence of the rate on the Earth's motion. This signal can be used to discriminate against background.

### 5.1. The non directional rate

The non directional differential rate is given by:

$$T(u) = \bar{R} a^2 |F(u)|^2 \Psi(a\sqrt{u}) , \quad T_{spin} = \bar{R}_{spin} a^2 |F_{11} u| \Psi(a\sqrt{u}) \quad (33)$$

where  $\bar{R}$  and  $\bar{R}_{spin}$  are the rates for the coherent and the spin contributions associated with some average LSP velocity. They carry the dependence on the SUSY parameters. They are

the most important ones, but they are not of interest in our present calculation.  $F(u)$  is the form factor, entering the coherent scattering and  $F_{11}(u)$  is the spin response function entering via the axial current (Vergados 2001b). The function  $\Psi$  depends on the LSP distribution velocity employed and is a function of the energy  $Q$  transferred to the nucleus

$$u = \frac{Q}{Q_0} \quad , \quad Q_0 = 4.1 \times 10^4 \text{ A}^{-4/3} \text{ KeV} \quad (34)$$

the parameter  $a$  is given by:

$$a = [\sqrt{2}\mu_r b v_0]^{-1} \quad (35)$$

where  $\mu_r$  is the reduced mass of the LSP-nucleus system and  $b$  is the (harmonic oscillation) size parameter.

The function, which is basic to us,  $\Psi$ , is given by

$$\Psi(x) = \int_0^x dy \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\phi y f(y, \theta, \phi) \quad (36)$$

with  $0 \leq x \leq (y_m - 1 + (\delta_1/2) \cos \alpha \sin \beta)$ .

The total rate is given by:

$$R = \int_{u_{min}}^{u_{max}} T(u) du \quad (37)$$

where  $u_{min}$  is determined from the cutoff energy of the detector and  $u_{max} = (y_m/a)^2$ . The dependence of the rate on the phase of the Earth is rather complicated. So we make use of the fact that the velocity of the Earth around the Sun is much smaller than the velocity of the Sun around the galaxy,  $\delta_1 \ll 1$ . So we can expand the previous expression into powers of  $\delta_1$  and, to leading order, put it in the form:

$$R = \bar{R}[R_0 + (R_1 \cos \alpha \sin \gamma - R_2 \cos \alpha \cos \gamma + R_3 \sin \alpha)\delta_1/2] \quad (38)$$

It turns out that the expansion coefficients  $R_2$  and  $R_3$  are zero. We can thus conveniently fit the rate with the formula:

$$R = \bar{R} t [1 + h \cos \alpha] \quad (39)$$

where  $h$  is the modulation amplitude (the difference between the maximum and the minimum is equal to  $2|h|$ ).

In the case of no modulation,  $\delta_1 = 0$ , the angular integrals can be done analytically to yield:

$$\Psi(x) = 2\pi N(\alpha_s, y_m) \sum_{n=3,5,7} A_n(\alpha_s, y_m) \frac{1}{n} J_n(x, y_m) \quad (40)$$

with  $0 \leq x \leq y_m - 1$  and

$$J_n(x, y_m) = J_{int}(n, y_m, x - 1) - J_{int}(n, y_m, x + 1) + 2 J_{int}(n, y_m, 1) \quad (41)$$

with  $A_n(\alpha_s, y_m) = (1 - (\alpha_s/2)y_m^2, \alpha_s/4, \alpha_s/4)$  and

$$J_{int}(n, y_m, y) = \int_0^y [y_m^2 - z^2]^{n/2} dz \quad (42)$$

The above integral can be done analytically to yield:

$$J_{int}(n, y_m, y) = yy_m^n {}_2F_1\left(\frac{1}{2}, \frac{-n}{2}, \frac{3}{2}, \frac{y^2}{y_m^2}\right) \quad (43)$$

where  ${}_2F_1$  is the usual hypergeometric function. For the cases of interest to us here the hypergeometric function can be simplified to yield:

$$J_{int}(3, y_m, y) = \frac{1}{8}[2y(y_m^2 - y^2)^{3/2} + 3y_m^2 y(y_m^2 y^2 - y^2)^{1/2} + 3y_m^4 \sin^{-1}(y/y_m)] \quad (44)$$

$$\begin{aligned} J_{int}(5, y_m, y) &= \frac{1}{24}[4y(y_m^2 - y^2)^{5/2} + 5y_m^2 y(y_m^2 - y^2)^{3/2} \\ &+ 15y_m^4 y(y_m^2 - y^2)^{1/2} + 15y_m^6 \sin^{-1}(y/y_m)] \end{aligned} \quad (45)$$

$$\begin{aligned} J_{int}(7, y_m, y) &= \frac{1}{192}[24y(y_m^2 - y^2)^{7/2} + 28y_m^2 y(y_m^2 - y^2)^{5/2} + 35y_m^4 y(y_m^2)^{1/2} \\ &+ 105y_m^6 y(y_m^2)^{1/2} + 105y_m^8 \sin^{-1}(y/y_m)] \end{aligned} \quad (46)$$

The corresponding expressions for the Gaussian expressions for  $\alpha_s$  cannot be done analytically. For the symmetric case,  $\alpha_s = 0$ , one finds that:

$$\Psi(x) = \frac{1}{2}[erf(x - 1) - erf(x + 1)] + 2 erf(1) \quad (47)$$

with  $erf(x)$  the error function:

$$erf(y) = \frac{2}{\pi} \int_0^y e^{-t^2} dt \quad (48)$$

The above functions  $\Psi$  are plotted in Figs 6 - 8.

We see that the velocity distribution does have a sizable effect on the total (non modulated rate). First we notice that the results depend on the asymmetry parameter  $\alpha_s$  in the case of the Eddington approach. This is not in agreement with the earlier results based on Gaussian distributions (Vergados 1999). In the present approach the restriction in the range

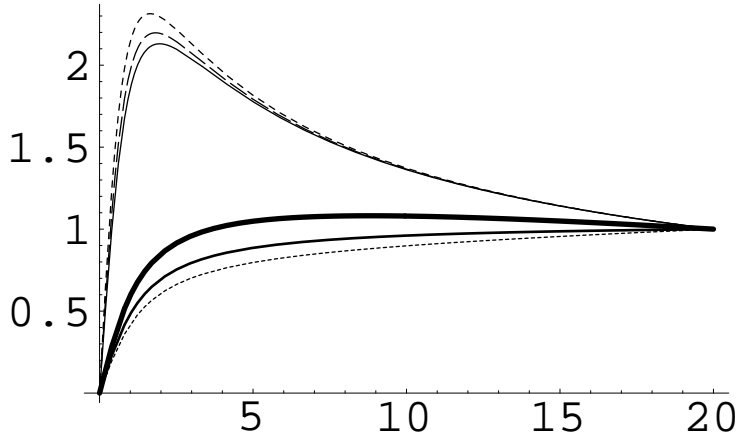


Fig. 5.— The same as in Fig. 3 for  $C_{mdm} = 0.5$ .

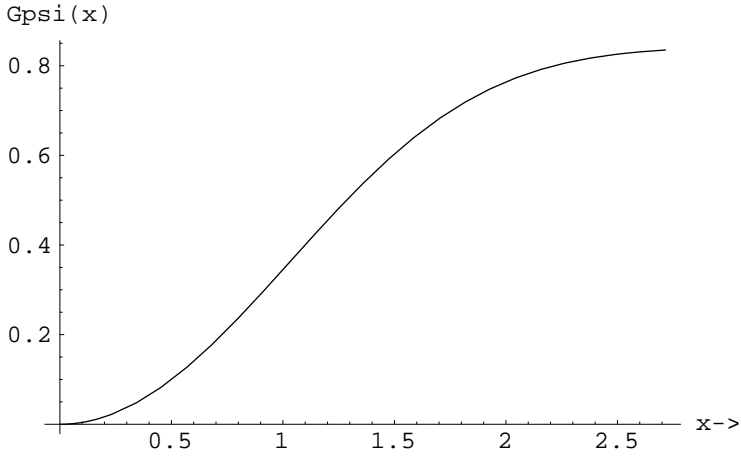


Fig. 6.— The function  $\Psi(x)$  for dark matter in the case of the symmetric Gaussian distribution.

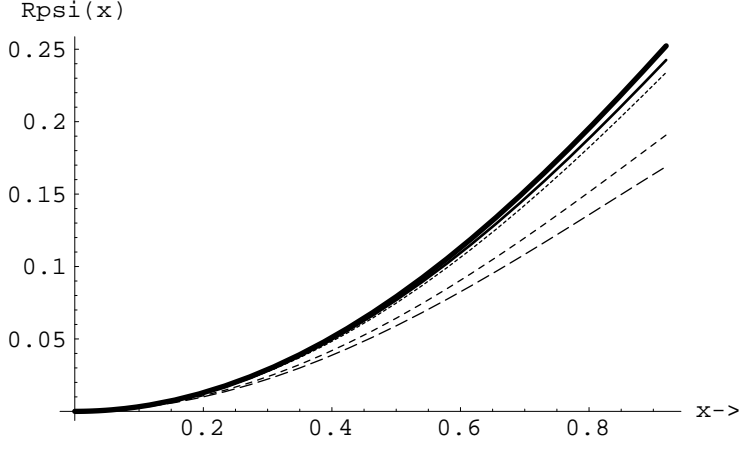


Fig. 7.— The function  $\Psi(x)$  for dark matter in the case of the Eddington theory for  $y_m = y_{esc}/\sqrt{2} = 1.92$ . The thick solid line, the fine solid line, the dotted line, the short dashing and the long dashing correspond to  $\alpha_s = -0.10, -0.05, 0, 0.40$  and  $0.80$  respectively.

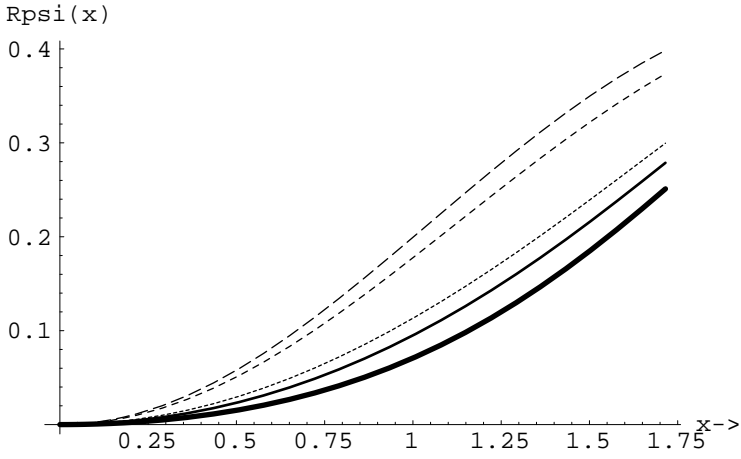


Fig. 8.— The function  $\Psi(x)$  for dark matter in the case of the Eddington theory for  $y_m = y_{esc} = 2.84$ .

of the available LSP velocities in the laboratory due to motion of the sun ( $y_m \rightarrow y_m - 1$ ) has much more dramatic consequences compared to the Gaussian case. Furthermore in the context Eddington theory  $y_m$  does not necessarily coincide with  $y_{esc}$ . All these effects combined lead to rates in the Eddington theory more suppressed compared to those of the Gaussian case. Especially in the high energy transfer regime and, in particular, when the detector energy cutoff is sizable. Effects like these may be more pronounced in the case of modulation, not studied in this work.

## 5.2. The directional rate

The directional differential rate is proportional to

$$\begin{aligned} T_{dir}(u) &= \frac{1}{2\pi} \bar{R} a^2 |F(u)|^2 \Psi_{dir}(a\sqrt{u}) \\ T_{dir}^{spin} &= \frac{1}{2\pi} \bar{R}_{spin} a^2 |F_{11}u| \Psi_{dir}(a\sqrt{u}) \end{aligned} \quad (49)$$

with

$$\Psi_{dir}(x) = \int_0^x dy \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi y f(y, \theta, \phi) X H(X) \quad (50)$$

with  $H[X]$  the well known Heaviside function and  $X$  is given by

$$X = \cos\Theta \cos\theta + \sin\Theta \sin\theta [\sin\Phi \sin\phi + \cos\Phi \cos\phi] \quad (51)$$

where  $\Theta$  and  $\Phi$  describe the direction of observation  $\hat{e}$

$$\hat{e} = \sin\Theta(\cos\Phi\hat{e}_x + \sin\Phi\hat{e}_y) + \cos\Theta\hat{e}_z$$

(There should be no confusion of the angle  $\Phi$  used here with the potential  $\Phi(r)$  used earlier). Note the presence of the factor of  $1/(2\pi)$ , since the azimuthal integration of the recoiling nucleus is not present and we intend to use the same nucleon cross-section both in the directional and the non directional case.

The total rate is proportional to:

$$R_{dir} = \int_{u_{min}}^{u_{max}} T_{dir}(u) du \quad (52)$$

Expanding again in powers of  $\delta_1$  we get an expression similar to Eq. 38. Thus, to leading order in  $\delta_1$ , we can fit the total rate by an expression of the form:

$$R_{dir} = \frac{1}{2\pi} \bar{R} t_{dir} [1 + (h_1 - h_2) \cos\alpha + h_3 \sin\alpha] \quad (53)$$



The parameters  $t_{dir}$  and  $h_i$   $i = 1, 2, 3$  are obviously functions of the direction of observation, i.e.  $\Theta$  and  $\Phi$ . If one observes in the direction of the Sun's velocity  $h_2 = h_3 = 0$ . Similarly if one observes in a plane perpendicular to the Sun's velocity  $h_1 = 0$ . Instead of  $t_{dir}$  it is best to use the ratio:

$$\kappa = 2\pi \frac{R_{dir}}{R} = \frac{t_{dir}}{t} \quad (54)$$

The parameter  $\kappa$  is essentially independent of the LSP mass, the nuclear parameters and the asymmetry parameter  $\alpha_s$ . But it depends strongly on the direction of observation and is expected to correlate strongly with the angle between  $\hat{e}$  and the Sun's direction of motion. This correlation provides an experimental signature perhaps better than the modulation with the Earth's motion in non directional experiments.

For  $\delta_1 \neq 0$  the above integrals over  $y, \theta, \phi$ , especially in the directional case, can only be done numerically. Such results will appear elsewhere (Braun et al. 2002).

## 6. Conclusions

In the present paper we studied the density and velocity distributions of cold dark matter in the context of the Eddington theory, considering not only symmetric but axially symmetric distributions as well.

We saw that in the context of this theory the predicted rotational velocities for the choice of potential  $n = 1$ , see Figs 3 and 4, can be fitted to the data, see e.g. Jungman *et al* (Jungman et al. 1996). The best choice seems to be the case with small asymmetry parameter  $\alpha_s$  and a relatively small fraction of ordinary matter to dark matter,  $C_{mdm} = 1$ . If the asymmetry parameter  $\alpha_s$  turns out to be negative, its absolute value is rather severely constrained from the rotational curves. We made no attempt to determine the absolute scale of the rotational curves and constrain the density distribution in our vicinity,  $\rho_0$ .

We have also studied the effect of the velocity distribution on the direct detection rates for cold dark matter. We have seen that, in the context of the Eddington approach, the total rates, unlike the case of the Gaussian distribution (Vergados 1999, 2000), somewhat sensitively depend on the asymmetry parameter  $\alpha_s$ .

Finally we should mention again that, in the Eddington approach, the escape velocity is not introduced ad hoc, but it comes in naturally. The distribution in this case automatically vanishes for velocities greater than a given velocity, which is determined from the gravitational potential. In the Gaussian case, Fig 8, however, the high velocity cut-off can only be put by hand.

Even if the allowed range of velocities is the same in both approaches, the total (non directional and non modulated) rates in our present approach are expected to be substantially smaller than those of the phenomenological Gaussian distributions (compare Figs 7 and 8). The effects of the distribution on the directional and/or modulated rates is currently under study, but we expect them to be a bit more pronounced than on the total rates.

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